

DO WE LIVE IN THE CENTER OF THE WORLD?

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ABSTRACT

We investigate the distribution of energy density in a stationary self-reproducing inflationary universe. We show that the main fraction of volume of the universe in a state with a given density ρ at any given moment of time t in synchronous coordinates is concentrated near the centers of deep exponentially wide spherically symmetric holes in the density distribution. A possible interpretation of this result is that a typical observer should see himself living in the center of the world. Validity of this interpretation depends on the choice of measure in quantum cosmology. Our investigation suggests that unexpected (from the point of view of inflation) observational data, such as possible local deviations from $\Omega = 1$, or possible dependence of the Hubble constant on the length scale, may tell us something important about quantum cosmology and particle physics at nearly Planckian densities.

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1. In this paper we are going to show that according to a very wide class of inflationary theories, the main fraction of volume of the universe in a state with a given density ρ at any given moment of time t (during or after inflation) should be concentrated near the centers of deep exponentially wide spherically symmetric holes in the density distribution.

Observational implications of this results depend on its interpretation. If we assume that we live in a part of the universe which is typical, and by “typical” we mean those parts of the universe which have the greatest volume with other parameters (time and density) being equal, then our result implies that we should live near the center of one of the holes in the density distribution. There should be many such holes in the universe, but each of them should be exponentially wide. Therefore an observer living near the center of any such hole will see himself “in the center of the world”.

One should clearly distinguish between the validity of our result and the validity of its interpretation suggested above. Even though the effect by itself is rather surprising (its existence was first conjectured in [1]), we think that we can confirm it by several different methods. Meanwhile the validity of its interpretation is much less clear. Until the interpretation problem is resolved in one way or another, there will remain an open possibility that inflationary cosmology predicts that we should live in a center of a spherically symmetric well. As we will see, this possibility is closely related to the question whether or not $\Omega = 1$ in inflationary universe.

2. It is well known that inflation makes the universe locally flat and homogeneous. On the other hand, during each time interval $H^{-1} = \sqrt{\frac{3}{8\pi V(\phi)}}$ the classical scalar field ϕ experiences quantum jumps of a typical amplitude $\delta\phi \sim \frac{H}{2\pi}$. (Here H is the Hubble constant; we use the system of units $M_p = 1$.) These jumps lead to small density perturbations with almost flat spectrum [2]. If we follow any particular inflationary domain, we can predict its behavior and calculate the typical amplitude of density perturbations in it.

However, this program can be accomplished by the methods of ref. [2] only if the value of the scalar field in the initial domain is small. Meanwhile if one starts with a domain containing a sufficiently large field ϕ , quantum fluctuations $\delta\phi$ become so large that they may give rise to an eternal process of self-reproduction of new inflationary domains. For example, in the theory $\frac{\lambda}{4}\phi^4$ the self-reproduction occurs in domains containing the field $\phi > \phi^* \sim \lambda^{-1/6}$ [3]. Such a domain very soon becomes divided into exponentially many domains containing all possible values of the scalar field (i.e. all possible values of its energy density) *at the same moment of time*. The knowledge of initial conditions in the original domain does not allow one to predict density perturbations or even mean density inside each of these new domains. Instead of that, one may try to study those domains which may look typical (i.e. most abundant) in the context of the new cosmological paradigm.

In particular, one may ask the following question. Suppose that we have one inflationary domain of initial size H^{-1} , containing scalar field $\phi > \phi^*$. Let us wait 15 billion years (in synchronous time t in each part of this domain) and see, what are the typical properties of those

parts of our original domain which at the present moment have density $10^{-29}\text{g}\cdot\text{cm}^{-3}$. As we will show, the answer to this question proves to be rather unexpected.

This domain exponentially expands, and becomes divided into many new domains of size H^{-1} , which evolve independently of each other. In many new domains the scalar field decreases because of classical rolling and quantum fluctuations. The rate of expansion of these domains rapidly decreases, and they give a relatively small contribution to the total volume of those parts of the universe which will have density $10^{-29}\text{g}\cdot\text{cm}^{-3}$ 15 billion years later. Meanwhile those domains where quantum jumps occur in the direction of growth of the field ϕ gradually push this field towards the upper bound where inflation can possibly exist, which is presumably close to the Planck boundary $V(\phi_p) \sim 1$. Such domains for a long time stay near the Planck boundary, and exponentially grow with the Planckian speed. Thus, the longer they stay near the Planck boundary, the greater contribution to the volume of the universe they give.

Those domains which 15 billion years later evolve into the regions with density $10^{-29}\text{g}\cdot\text{cm}^{-3}$ cannot stay near the Planck boundary for indefinitely long time. However, they will do their best if they stay there as long as it is possible. In fact they will do even better if they stay near the Planck boundary even longer, and then rush down with the speed exceeding the speed of classical rolling. This may happen if quantum fluctuations coherently add up to large quantum jumps towards small ϕ . This process in some sense is dual to the process of perpetual climbing up, which leads to the self-reproduction of inflationary universe.

Of course, the probability of large quantum jumps down is exponentially suppressed. However, by staying longer near the Planck boundary inflationary domains get an exponentially large contribution to their volume. These two exponential factors compete with each other to give us an optimal trajectory by which the scalar field rushes down in those domains which eventually give the leading contribution to the volume of the regions of a given density ρ at a given time t . From what we are saying it should be clear that the quantum jumps of the scalar field along such optimal trajectories should have a greater amplitude than its naively estimated value $\frac{H}{2\pi}$, and they should preferably occur in the downwards direction. As a result, the energy density along these optimal trajectories will be smaller than the energy density of their lazy neighbors which prefer to slide down without too much of jumping. This creates holes in the distribution of energy density. We are going to show that at any given time t most of the volume of the regions of the universe containing matter of any given density ρ lies very close to the centers of these holes.

3. The best way to examine this scenario is to investigate the probability distribution $P_p(\phi, t)$ to find a domain of a given physical volume (that is why we use notation P_p) in a state with a given field ϕ at some moment of time t . The probability distribution $P_p(\phi, t)$ obeys the following diffusion equation (see [1] and the references therein):

$$\frac{\partial P_p}{\partial t} = \frac{1}{2} \frac{\partial}{\partial \phi} \left(\frac{H^{3/2}(\phi)}{2\pi} \frac{\partial}{\partial \phi} \left(\frac{H^{3/2}(\phi)}{2\pi} P_p \right) + \frac{V'(\phi)}{3H(\phi)} P_p \right) + 3H(\phi) P_p . \quad (1)$$

Note, that this equation is valid only during inflation, which typically occurs within some limited interval of values of the field ϕ : $\phi_{min} < \phi < \phi_{max}$. In the simplest versions of chaotic inflation model $\phi_{min} \sim 1$, and ϕ_{max} is perhaps close to the Planck boundary ϕ_p , where $V(\phi_p) = 1$. To find solutions of this equation one must specify boundary conditions. Behavior of solutions typically is not very sensitive to the boundary conditions at the end of inflation at ϕ_{min} ; it is sufficient to assume that the diffusion coefficient (and, correspondingly, the double derivative term in the r.h.s. of equation (1)) vanishes for $\phi < \phi_{min}$. The conditions near the Planck boundary $\phi = \phi_p$ play a more important role. In this paper we will assume, following [1], that there can be no inflation at $V(\phi) > 1$, and therefore we will impose on P_p the simplest boundary condition $P_p(\phi, t)|_{\phi > \phi_p} = 0$. At the end of the paper we will discuss possible modifications of our results if other boundary conditions should be imposed at the upper boundary of the region of values of the field ϕ where inflation is possible.

One may try to obtain solutions of equation (1) in the form of the series $P_p(\phi, t) = \sum_{s=1}^{\infty} e^{\lambda_s t} \pi_s(\phi)$. In the limit of large time t only the term with the largest eigenvalue λ_1 survives, $P_p(\phi, t) = e^{\lambda_1 t} \pi_1(\phi)$. The function π_1 in the limit $t \rightarrow \infty$ will have a meaning of a normalized *time-independent* probability distribution to find a given field ϕ in a unit physical volume, whereas the function $e^{\lambda_1 t}$ shows the overall growth of the volume of all parts of the universe, which does not depend on ϕ in the limit $t \rightarrow \infty$. In this limit one can write eq. (1) in the form

$$\frac{1}{2} \frac{\partial}{\partial \phi} \left(\frac{H^{3/2}(\phi)}{2\pi} \frac{\partial}{\partial \phi} \left(\frac{H^{3/2}(\phi)}{2\pi} \pi_1(\phi) \right) \right) + \frac{\partial}{\partial \phi} \left(\frac{V'(\phi)}{3H(\phi)} \pi_1(\phi) \right) + 3H(\phi) \cdot \pi_1(\phi) = \lambda_1 \pi_1(\phi) . \quad (2)$$

In this paper we will concentrate on the simplest theory with $V(\phi) = \frac{\lambda}{4} \phi^4$, $H = \sqrt{\frac{2\pi\lambda}{3}} \phi^2$. Eq. (2) for this theory reads [1]:

$$\pi_1'' + \pi_1' \left(\frac{6}{\lambda \phi^5} + \frac{9}{\phi} \right) + \pi_1 \left(\frac{6}{\lambda \phi^6} + \frac{15}{\phi^2} + \frac{36\pi}{\lambda \phi^4} - \frac{\lambda_1}{\pi \phi^6} \left(\frac{6\pi}{\lambda} \right)^{3/2} \right) = 0 . \quad (3)$$

We have solved this equation both analytically and numerically, and have found that the eigenvalue λ_1 is given by $d(\lambda) H_{max}$, where $d(\lambda)$ is the fractal dimension which approaches 3 in the limit $\lambda \rightarrow 0$, and H_{max} is the maximal value of the Hubble constant in the allowed interval of ϕ . In our case $H_{max} = H(\phi_p) = 2\sqrt{\frac{2\pi}{3}}$. Thus, in the small λ limit one has $\lambda_1 = 3H(\phi_p) = 2\sqrt{6\pi} \approx 8.68$.

Note, that the distribution π_1 depends on ϕ very sharply. For example, one can easily check that at small ϕ (at $\phi \lesssim \lambda^{-1/8} \lambda_1^{-1/4}$) the leading terms in eq. (3) are the second and the last ones⁴. Therefore the solution of equation (3) at $\phi \lesssim \lambda^{-1/8} \lambda_1^{-1/4}$ is given by

$$\pi_1 \sim \phi \sqrt{\frac{6\pi}{\lambda}} \lambda_1 . \quad (4)$$

This is an extremely strong dependence. For example, $\pi_1 \sim \phi^{1.2 \cdot 10^8}$ for the realistic value $\lambda \sim 10^{-13}$. All surprising results we are going to obtain are rooted in this effect. One of the

⁴The appearance of the new scale $\phi \sim \lambda^{-1/8}$ in the theory $\lambda \phi^4$ is explained in [4], and was also pointed out by Mukhanov (private communication).

consequences is the distribution of energy density ρ . For example, during inflation $\rho \approx \frac{\lambda}{4}\phi^4$. This implies that the distribution of domains of density ρ is $P_p(\rho) \sim \rho^{3 \cdot 10^7}$. Thus at each moment of time t the self-reproducing inflationary universe consists of indefinitely large number of domains containing matter with all possible values of density, the total volume of all domains with density 2ρ being approximately 10^{10^7} times greater than the total volume of all domains with density ρ ! If this result is understood (see its discussion in [1]), all the rest should be easy...

Let us consider now all parts of inflationary universe which contain a given field ϕ at a given moment of time t . One may wonder, what was the value of this field in those domains at the moment $t - H^{-1}$? The answer is simple: One should add to ϕ the value of its classical drift $\dot{\phi}H^{-1}$. One should also add the amplitude of a quantum jump $\Delta\phi$. The typical jump is given by $\delta\phi = \pm \frac{H}{2\pi}$. At the last stages of inflation this quantity is by many orders of magnitude smaller than $\dot{\phi}H^{-1}$. But in which sense jumps $\pm \frac{H}{2\pi}$ are typical? If we consider any particular initial value of the field ϕ , then the typical jump from this point is indeed given by $\pm \frac{H}{2\pi}$. However, if we are considering all domains with a given ϕ and trying to find all those domains from which the field ϕ could originate back in time, the answer may be quite different. Indeed, the total volume of all domains with a given field ϕ at any moment of time t strongly depends on ϕ : $P_p(\phi) \sim \pi_1(\phi) \sim \phi^{\sqrt{\frac{6\pi}{\lambda}}\lambda_1} \sim \phi^{10^8}$, see eq. (4). This means that the total volume of all domains which could jump towards the given field ϕ from the value $\phi + \Delta\phi$ will be enhanced by a large additional factor $\frac{P_p(\phi+\Delta\phi)}{P_p(\phi)} \sim \left(1 + \frac{\Delta\phi}{\phi}\right)^{\sqrt{\frac{6\pi}{\lambda}}\lambda_1}$. On the other hand, the probability of large jumps $\Delta\phi$ is suppressed by the Gaussian factor $\exp\left(-\frac{2\pi^2\Delta\phi^2}{H^2}\right)$. One can easily verify that the product of these two factors has a sharp maximum at $\Delta\phi = \lambda_1\phi \cdot \frac{H}{2\pi}$, and the width of this maximum is of the order $\frac{H}{2\pi}$. In other words, most of the domains of a given field ϕ are formed due to jumps which are greater than the “typical” ones by a factor $\lambda_1\phi \pm 1$.

4. This result is very unusual. We became partially satisfied by our understanding of this effect only after we confirmed its existence by four different methods, including computer simulations. A detailed description of our results will be contained in a separate publication [5]. Here we will briefly describe the computer simulations which we have performed.

We have studied a set of domains of initial size H^{-1} filled with large homogeneous field ϕ . We considered large initial values of ϕ , where the self-reproduction of inflationary domains is possible. From the point of view of stochastic processes which we study, each domain can be modelled by a single point with the field ϕ in it. Our purpose was to study the typical amplitude of quantum jumps of the scalar field ϕ in those domains which reached some value ϕ_0 close to the end of inflation.

Each step of our calculations corresponds to a time change $\Delta t = uH_0^{-1}$. Here $H_0 \equiv H(\phi_0)$, and u is some number, $u < 1$. The results do not depend on u if it is small enough. The evolution of the field ϕ in each domain consists of several independent parts. First of all, the field evolves according to classical equations of motion during inflation. Secondly, it makes quantum

jumps by $\delta\phi = \frac{H}{\pi}\sqrt{\frac{uH}{2H_0}} \sin r_i$. Here r_i is a set of random numbers, which are different for each inflationary domain. To account for the growth of physical volume of each domain we used the following procedure. We followed each domain until its radius grows two times, and after that we considered it as 8 independent domains. If we would continue doing so for a long time, the number of such domains (and our distribution $P_p(\phi, t)$) would become exponentially large. We will describe in [5] the procedure which we used to overcome this difficulty. In accordance with our condition $P_p(\phi, t)|_{\phi > \phi_p} = 0$, we removed all domains where the field ϕ jumped to the super-Planckian densities $V(\phi) > 1$. After a sufficiently large time t the distribution of domains followed by the computer with a good accuracy approached the stationary distribution π_1 which we have obtained in [1] by a completely different method. We used it as a consistency check for our calculations.

We kept in the computer memory information about all jumps of all domains during the last time interval H_0^{-1} . This made it possible to evaluate an average sum of all jumps of those domains in which the scalar field became smaller than some value ϕ within the last time interval H_0^{-1} . Naively, one could expect this value to be smaller than $\frac{H_0}{2\pi}$, since the average amplitude of the jump is $\frac{H_0}{2\pi}$, but they occur both in the positive and negative directions. However, our simulations confirmed our analytical result $\Delta\phi = \lambda_1\phi \cdot \frac{H_0}{2\pi}$. In other words, we have found that most of the domains which reach the hypersurface $\phi = \phi_0$ within a time interval $\Delta t = H_0^{-1}$ do it by rolling accompanied by persistent jumps down, which have a combined amplitude $\lambda_1\phi_0$ times greater than $\frac{H_0}{2\pi}$.

Even though we obtained this result in the context of the theory $\frac{\lambda}{4}\phi^4$, it can be easily generalized. One can show that for any inflationary theory in the regime where one can neglect the first (diffusion) term in the l.h.s. of equation (2) (which is usually possible at the last stages of inflation), most of the domains of a given field ϕ are formed due to jumps which are greater than the “typical” ones by a factor $\lambda_1 \frac{4V(\phi)}{V'(\phi)}$ [5].

5. As we already mentioned, the probability of large fluctuations should be suppressed by the factor $\exp\left(-\frac{2\pi^2\delta\phi^2}{H^2}\right)$, which in our case gives the suppression factor $\sim \exp(-10^3)$. It is well known that exponentially suppressed perturbations typically give rise to spherically symmetric bubbles [6]. Note also, that the Gaussian distribution suppressing the amplitude of the perturbations refers to the amplitude of a perturbation in its maximum. Let us show now that the main part of the volume of the universe in a state with a given ϕ (or with a given density ρ) corresponds to the centers of these bubbles.

Consider again the part of the universe with a given ϕ at a given time t . We have found that most of the jumps producing this field ϕ during the previous time interval H^{-1} occurred from a very narrow region $\phi + \frac{\dot{\phi}}{H} + \lambda_1\phi\frac{H}{2\pi}$ of values of the scalar field. The width of this region was found to be of the order of $\frac{H}{2\pi}$, which is much smaller than the typical depth of our bubble $\Delta\phi \sim \lambda_1\phi\frac{H}{2\pi}$. Now suppose that the domain containing the field ϕ appears not at the center of the bubble, but at its wall. This would mean that the field near the center of the bubble is somewhat smaller than ϕ . Such a configuration could be created by a jump from $\phi + \frac{\dot{\phi}}{H} + \lambda_1\phi\frac{H}{2\pi}$ only if the amplitude

of the jump is greater than $\lambda_1 \phi \frac{H}{2\pi}$. However, we have found that the main contribution to the volume of domains with a given ϕ is produced by jumps of an amplitude $\lambda_1 \phi \frac{H}{2\pi} \pm \frac{H}{2\pi}$, the greater deviation from the typical amplitude $\lambda_1 \phi \frac{H}{2\pi}$ being exponentially suppressed. This means that the scalar field ϕ can differ from its value at the center of the bubble by no more than the usual amplitude of scalar field perturbations $\frac{H}{2\pi}$, which is smaller than the depth of the bubble by a factor $(\lambda_1 \phi)^{-1}$. Thus, the main fraction of the volume of the universe with a given ϕ (or with a given density of matter) can be only slightly outside the center, which may lead to a small dipole anisotropy of the microwave background radiation.

We should emphasize that our results are based on the investigation of the global structure of the universe rather than of the structure of each particular bubble. If one neglects that the universe is a fractal and looks only at one particular bubble (i.e. at the one in which we live now), then one can easily see that inside each bubble there is a plenty of space far away from its center. Therefore one could conclude that there is nothing special about the centers of the bubbles. However, when determining the fraction of domains near the centers we were comparing the volumes of *all* regions of *equal* density at equal time. Meanwhile, the density ρ_{wall} of matter on the walls of a bubble is greater than the density ρ_{center} in its center. As we have emphasized in the discussion after eq. (4), the total volume of *all* domains of density ρ_{wall} is greater than the total volume of all domains of density ρ_{center} by the factor $(\rho_{\text{wall}}/\rho_{\text{center}})^{3 \cdot 10^7}$. Thus, the volume of space outside the center of the bubble is not smaller than the space near the center. However, going outside the center brings us to the region of a different density, $\rho_{\text{wall}} > \rho_{\text{center}}$. Our results imply that one can find much more space with $\rho = \rho_{\text{wall}}$ not at the walls of our bubble, but near the centers of *other* bubbles.

6. The nonperturbative jumps down should occur on all scales independently. At the earlier stage, when the time changed from $t - 2H^{-1}$ to $t - H^{-1}$, the leading contribution to the volume of the universe of a given density should be also given by domains which are jumping down by $\Delta\phi \sim \lambda_1 \phi \frac{H}{2\pi}$. This implies that on each new length scale different from the previous one by the factor of e (one new e-folding) our trajectory runs down creating a local depression of the scalar field equal to $\lambda_1 \phi \frac{H}{2\pi}$. One may visualize the resulting distribution of the scalar field in the following way. At some scale r the deviation of the field ϕ from homogeneity can be approximately represented as a hole of a radius r with the depth $\lambda_1 \phi \frac{H}{2\pi}$. Near the bottom of this hole there is another hole of a smaller radius $e^{-1}r$ and approximately of the same depth $\lambda_1 \phi \frac{H}{2\pi}$. Near the center of this hole there is another hole of a radius $e^{-2}r$, etc. Of course, this is just a discrete model. The shape of the smooth distribution of the scalar field is determined by the equation

$$\frac{d\phi}{d \ln r H} = \lambda_1 \phi \frac{H}{2\pi} = \sqrt{\frac{\lambda}{6\pi}} \lambda_1 \phi^3, \quad (5)$$

which gives

$$\phi^2(r) \approx \frac{\phi^2(0)}{1 - \lambda_1 \phi^2(0) \sqrt{\frac{2\lambda}{3\pi}} \ln r H} \quad \text{for } r > H^{-1}. \quad (6)$$

Note that $\phi(r) \approx \phi(0)$ for $r < H^{-1}$ (there are no perturbations of the classical field on this scale).

This distribution is altered by the usual small perturbations of the scalar field. These perturbations are responsible for the large nonperturbative jumps down when they add up near the center of the hole, but at a distance much greater than their wavelength from the center of the hole these perturbations have the usual magnitude $\frac{H}{2\pi}$. Thus, our results do not lead to considerable modifications of the usual density perturbations which lead to galaxy formation. However, the presence of the deep hole (6) can significantly change the local geometry of the universe.

In the inflationary scenario with $V(\phi) = \frac{\lambda}{4}\phi^4$ fluctuations which presently have the scale comparable with the horizon radius $r_h \sim 10^{28}$ cm have been formed at $\phi \sim 5$ (in the units $M_p = 1$). As we have mentioned already $\lambda_1 \approx 2\sqrt{6\pi} \sim 8.68$ for our choice of boundary conditions [1]. This means that the jump down on the scale of the present horizon should be $\lambda_1\phi \sim 40$ times greater than the standard jump. In the theory $\frac{\lambda}{4}\phi^4$ the standard jumps lead to density perturbations of the amplitude $\frac{\delta\rho}{\rho} \sim \frac{2\sqrt{6\lambda\pi}}{5}\phi^3 \sim 5 \cdot 10^{-5}$ (in the normalization of [7]). Thus, according to our analysis, the nonperturbative decrease of density on each length scale different from the previous one by the factor e should be about $\frac{\delta\rho}{\rho} \sim \lambda_1 \frac{2\sqrt{6\lambda\pi}}{5}\phi^4 \sim 2 \cdot 10^{-3}$. This allows one to evaluate the shape of the resulting hole in the density distribution as a function of the distance from its center. One can write the following equation for the scale dependence of density:

$$\frac{1}{\rho} \frac{d\rho}{d \ln \frac{r}{r_0}} = -\lambda_1 \cdot \frac{2\sqrt{6\lambda\pi}}{5} \phi^4, \quad (7)$$

where r is the distance from the center of the hole. Note that $\phi = \frac{1}{\sqrt{\pi}} (\ln \frac{r}{r_0})^{\frac{1}{2}}$ in the theory $\frac{\lambda}{4}\phi^4$ [7]. Here r_0 corresponds to the smallest scale at which inflationary perturbations have been produced. This scale is model-dependent, but typically at present it is about 1 cm. This yields

$$\frac{\Delta\rho}{\rho_c} \equiv \frac{\rho(r) - \rho(r_0)}{\rho(r_0)} = \frac{2\lambda_1\sqrt{2\lambda}}{5\pi\sqrt{3\pi}} \ln^3 \frac{r}{r_0}. \quad (8)$$

This gives the typical deviation of the density on the scale of the horizon (where $\ln \frac{r_h}{r_0} \sim 60$) from the density at the center: $\frac{\Delta\rho}{\rho_c} \sim 750 \cdot \frac{\delta\rho}{\rho} \sim 4 \cdot 10^{-2}$.

7. It is very tempting to interpret this effect in such a way that the universe around us becomes locally open, with $1 - \Omega \sim 10^{-1}$. Indeed, our effect is very similar to the one discussed in [8, 9], where it was shown that the universe becomes open if it is contained in the interior of a bubble created by the $O(4)$ symmetric tunneling. Our nonperturbative jumps look very similar to tunneling with the bubble formation. However, unlike in the case considered in [8, 9], our bubbles appear on all length scales. This removes the problem of fine-tuning, which plagues the one-bubble models of the universe [9]. However, it rises another problem, which may be even more complicated.

Note, that the results discussed above refer to the density distribution at the moment when the corresponding wavelengths were entering horizon. At the later stages gravitational instability should lead to growth of the corresponding density perturbations. Indeed, we know that density

perturbations on the galaxy scale have grown more than 10^4 times in the linear growth regime until they reached the amplitude $\frac{\delta\rho}{\rho} \sim 1$, and then continued growing even further. The same can be expected in our case, but even in a more dramatic way since our “density perturbations” on all scales are about 40 times greater than the usual density perturbations which are responsible for galaxy formation. This would make the center of the hole very deep; its density should be many orders of magnitude smaller than the density of the universe on the scale of horizon. Moreover, the center would be devoid of any structures necessary for the existence of our life. Indeed, on each particular scale the jump down completely overwhelms the amplitude of usual density perturbations. The bubble cannot contain any galaxies at the distance from the center comparable with the galaxy scale, it cannot contain any clusters at the distance comparable with the size of a cluster, etc.

This problem can be easily resolved. Indeed, our effect (but not the amplitude of the usual density perturbations) is proportional to λ_1 , which is determined by the maximal value of the Hubble constant compatible with inflation. If, for example, the maximal energy scale in quantum gravity or in string theory is given not by 10^{19} GeV, but by 10^{18} GeV, then the parameter λ_1 will decrease by a factor 10^{-2} . Other ways to change λ_1 are described in [10]. Thus it is easy to make our effect very small without disturbing the standard predictions of inflationary cosmology.

It is possible though that we will not have any problems even with a large λ_1 if we interpret our results more carefully. An implicit hypothesis behind our interpretation is that we are typical, and therefore we live and make observations in those parts of the universe where most other people do. One may argue that the total number of observers which can live in domains with given properties (e.g. in domains with a given density) should be proportional to the total volume of these domains at a given time. However, our existence is determined not by the local density of the universe but by the possibility for life to evolve for about 5 billion years on a planet of our type in a vicinity of a star of the type of the Sun. If, for example, we have density $10^{-29}\text{g}\cdot\text{cm}^{-3}$ inside the center of the hole, and density $10^{-27}\text{g}\cdot\text{cm}^{-3}$ on the horizon, then the age of our part of the universe (or, to be more accurate, the time after the end of inflation) will be determined by the average density $10^{-27}\text{g}\cdot\text{cm}^{-3}$, and it will be smaller than 5 billion years. Moreover, as we have mentioned above, any structures such as galaxies or clusters cannot be formed near the centers of the holes.

Thus, the naive idea that the number of observers is proportional to volume does not work at the distances from the centers which are much smaller than the present size of the horizon. Even though at any given moment of time most of the volume of the universe at the density $10^{-29}\text{g}\cdot\text{cm}^{-3}$ is concentrated near these holes, the corresponding parts of the universe are too young and do not have any structures necessary for our existence. Volume alone does not mean much. We live on the surface of the Earth even though the volume of empty space around us is incomparably greater.

On the other hand, the naive relation between the number of observers and the volume in a state of a given density may work nicely for the matter distribution on the scale of the present horizon. These considerations may make the distribution (8) homogeneous near the center not up

to the scale $r_0 \sim 1$ cm, but up to some much greater scale $r \gg 10^{22}$ cm. The resulting distribution may appear relatively smooth. It may resemble an open universe with a scale-dependent effective parameter $\Omega(r)$.

An additional ambiguity in our interpretation appears due to the dependence of the distribution P_p on the choice of time parametrization. Indeed, there are many different ways to define “time” in general relativity. If, for example, one measures time not by clock but by rulers and determines time by the degree of a local expansion of the universe, then in this “time” the rate of expansion of the universe does not depend on its density. As a result, our effect is absent in this time parametrization [1]. In this paper we used the standard time parametrization which is most closely related to our own nature (time measured by number of oscillations rather than by the distance to the nearby galaxies). But maybe we should use another time parametrization, or even integrate over all possible time parametrizations? The last possibility, suggested to us by Robert Wagoner, is very interesting, since after the integration we would obtain a parametrization-independent measure in quantum cosmology. Even more radical possibility is the integration over all possible values of time. However, the corresponding integral diverges in the limit $t \rightarrow \infty$ [10]. Right now we still do not know what is the right way to go. We do not even know if it is right that we are typical and that we should live in domains of the greatest volume, see the discussion of this problem in [1, 10].

Until the interpretation problem is resolved, it will remain unclear whether our result is just a mathematical curiosity similar to the twin paradox, or it can be considered as a real prediction of properties of our part of the universe. On the other hand, at present we cannot exclude the last possibility, and this by itself is a very unexpected conclusion. Few years ago we would say that the possibility that we live in a local “center of the world” definitely contradicts inflationary cosmology. Now we can only say that it is an open question to be studied both theoretically and experimentally. As we have seen, the relation of our result to the properties of our part of the universe depends on interpretation of quantum cosmology, and the amplitude of our effect is very sensitive to the properties of the universe at nearly Planckian densities. This suggests that unexpected (from the point of view of inflation) observational data, such as possible local deviations from $\Omega = 1$, or possible dependence of the Hubble constant on the length scale, may tell us something important about quantum cosmology and particle physics near the Planck density.

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